Enrollment No: _____ **Exam Seat No:**_____

C.U.SHAH UNIVERSITY
Winter Examination-2015

Subject Name : Discrete Mathematics

Subject Code : 4TE04DSM1 **Branch :** Computer Engineering / Information Technology

Semester : 4 **Date :** 18/11/2015 **Time :** 2:30 To 5:30 **Marks :** 70

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions (2 Marks each)** **(14)**

a) How many edges are there in undirected graph with 5 vertices each of degree 4?

b) Define Cyclic group and give an example of it.

c) Construct the truth table for $(P \to Q) \wedge (Q \to P)$.

d) Find the least and greatest element in the POSET $\langle Z^+, D \rangle$, if they exist.

e) Define: Atom and find all atoms of Boolean algebra $\langle S_{30}, D \rangle$.

f) Symbolize the expression:
i) "If x is odd and x is perfect square then x is divisible by 3."
ii) " The crop will be destroyed if there is a flood"

g) Give an example of a semi group which is not a monoid. Justify your answer.

Attempt any four questions from Q-2 to Q-8

Q-2 State and prove Stone's representation theorem. **(14)**

Q-3 **Attempt all questions** **(14)**

A Obtain the Sum of product canonical form of the Boolean expression in three variables $(x_1 * x_3) \oplus (x_1' * x_2) \oplus (x_2 * x_3)$.

B Define Kernal of Homomorphism and hence show that the kernel of homomorphism $g: \langle G, * \rangle \to \langle H, \Delta \rangle$ is a subgroup of a group $\langle G, * \rangle$.

Q-4 **Attempt all questions** **(14)**

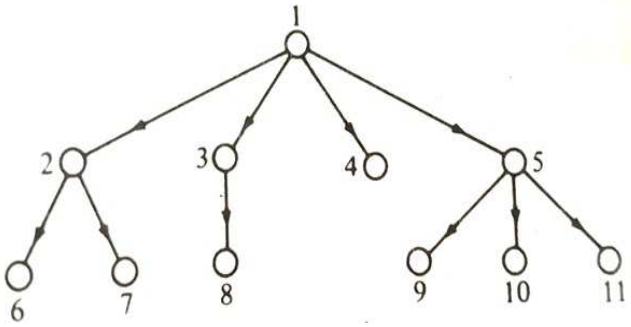
A For the poset $\langle \{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\}, \subseteq \rangle$,
1) Draw the Hasse diagram.
2) Find maximal elements and minimal elements
3) Find Greatest element and least element, if exists
4) Find Lower bounds of {1,3,4} and {2,3,4}
5) Find Upper bounds of {2,4} and {3,4}

B Let (L , ≤) be a lattice in which $*$ and $\oplus$ denote operations of meet and join. Then for any a, b ∈ L prove that $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

Q-5 **Attempt all questions** **(14)**

A State and prove Cayley's theorem on group.

B Obtain binary tree equivalent to the tree given below:



Q-6 **Attempt all questions** **(14)**

A Let $S=\{1,2,3\}$ and S_3 be the set of permutation on S. Find all proper subgroups of a group $\langle S_3, \Diamond \rangle$ and identify that which subgroup is normal? Where $\Diamond$ represents composition of two permutations.

B Let (B, $*$, $\oplus$, ', 0, 1) be a Boolean algebra prove that
$a = b \Leftrightarrow (a * b') \oplus (a' * b) = 0$

Q-7 **Attempt all questions** **(14)**

A Show that $\langle Z_6, +_6 \rangle$ is isomorphic to $\langle Z_7^*, \times_7 \rangle$.

B Let $E=\{a,b,c,d,e\}$, $\underset{\sim}{A}=\{(a,0.3),(b,0.8),(c,0.5),(d,0.1),(e,0.9)\}$

$\underset{\sim}{B}=\{(a,0.7),(b,0.6),(c,0.4),(d,0.2),(e,0.1)\}$

Find: (1) $\underset{\sim}{A}\cup\underset{\sim}{B}$ (2) $\underset{\sim}{A}\cdot\underset{\sim}{B}$ (3) $\underset{\sim}{A}\hat{+}\underset{\sim}{B}$ (4) $\underset{\sim}{A}-\underset{\sim}{B}$ (5) Verify any one De Morgan's Law

Q-8 **Attempt all questions** **(14)**

A State and prove Lagrange's theorem on group.

B From the graph given below, answer the following:

1. Find in degree, out degree and total degree of each vertex.
2. Find reachable set of each vertex.
3. Find all node bases.
4. Find all strong components.
5. Write the adjacency matrix from the given digraph.